Maximization of Time-to-First-failure for Multicast Applications with Omnidirectional Antennas

Arindam K. Das, Robert J. Marks, Mohamed El-Sharkawi, Payman Arabshahi, Andrew Gray

Abstract—We consider the problem of maximizing the time-to-first-failure (TTFF), defined as the time till the first node in the network runs out of battery energy, in energy constrained broadcast wireless networks. We show that the TTFF criterion, by itself, fails to provide the "ideally optimum" tree and propose a composite weighted objective function which maximizes the TTFF and minimizes the sum of transmitter powers. We then develop a mixed integer linear programming (MILP) model for solving the joint optimization problem optimally. We also consider the case of prioritized nodes and show how the model can be modified to deal with such priorities.

I. Introduction

We consider the problem of maximizing the time-to-first-failure in energy constrained broadcast wireless networks where each node is powered by batteries. In applications where replacement/maintenance of such batteries is difficult or infeasible, it is of utmost importance to design routing protocols which maximize the $lifetime$ of the network. A metric commonly used to define the lifetime of a network is the duration of time before any node in the network runs out of its battery energy. We define this time to be the $time-to-first-failure$ (TTFF), also known as $system\ lifetime$ or $network\ lifetime$ in the literature. To the best of our knowledge, this problem was first addressed by Chang and Tassiulas for an unicast application [1]. Subsequent research in this area for unicast as well as multicast applications have been reported in [2], [3], [4] and [5]. In [8], it is shown that maximization of the TTFF for a broadcast application can be solved optimally by a greedy algorithm in polynomial time.

In this paper, we first illustrate with an example that simply optimizing the TTFF criterion may not provide the best possible solution. This motivates the use of a composite objective function involving the the sum of the transmitter powers. We then present a mixed integer linear programming (MILP) model for solving the joint optimization problem optimally. The MILP model is based on the well-studied single-origin multiple-destination uncapacitated flow problem, tailored to reflect the inherently broadcast nature of the wireless medium. Finally, we consider the case of prioritized nodes and show how the model can be modified to deal with such priorities.

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II. Network Model

We assume a fixed N-node network with a specified source node which has to broadcast a message to all other nodes in the network. Any node can be used as a relay node to reach other nodes in the network. All nodes are assumed to have omni-directional antennas, so that if node i transmits to node j, all nodes closer to i than j will also receive the transmission (provided line-of-sight exists).

We assume that, for a transmission from node i to j, the received signal power at j varies as $d_{ij}^{-\alpha}$, where

$$d_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2}$$

is the Euclidean distance between nodes i and j, $\{(x_i, y_i)\}$ are the coordinates of node i and α (typically in the range $2 \leq \alpha \leq 4$) is the channel loss exponent. Consequently, the transmitter power at i necessary to support the link $i \rightarrow j$, $\mathbf{P}_{ij}$, is proportional (accounting for fading and antenna gain factors) to d_{ij}^{α}. Without any loss of generality, we set the proportionality constant to be equal to 1 and therefore:

$$\mathbf{P}_{ij} = d_{ij}^{\alpha} \tag{1}$$

The power matrix of a network, $\mathbf{P}$, is defined to be an $N \times N$ symmetric matrix whose $(i, j)th$ element represents the power required to support the link $i \rightarrow j$, $\mathbf{P}_{ij}$.

Finally, we assume that power expenditures due to signal reception and processing are negligible compared to signal transmission and hence the lifetime is determined solely by the choice of transmitter powers and residual energy levels of the nodes.

III. Problem Statement

Let $E(t)$ be a vector of node residual energies at time t, the ith element of $E(t)$ representing the residual energy of node i at time t, and Y be a vector of node transmission powers. The element Y_i represents the transmitter power level of node i. We assume that each node has a constraint on maximum transmitter power, denoted by Y_i^{max}. That is:

$$Y_i \leq Y_i^{max} : \ \forall i \in \mathcal{N} \tag{2}$$

where $\mathcal{N}$ is the set of all nodes in the network.

Also, let $\mathcal{D}$ the set of destination nodes and $\mathcal{E}$ the set of all directed edges[1] and $\mathcal{D}$ the set of destination nodes,

[1] In this paper, we assume that all edges are directed. The notation $(i \rightarrow j)$ will be used to denote a directed edge from node i to j. The notation (i, j) will be used to refer to the node pair.

$\mathcal{D} \subseteq \{\mathcal{N} \setminus source\}$. Let the cardinality of these sets be N, E and D respectively; *i.e.*, $N = |\mathcal{N}|$, $E = |\mathcal{E}|$ and $D = |\mathcal{D}|$. Using the transmitter power constraint, the set of all edges, $\mathcal{E}$, is given by:

$$\mathcal{E} = \{(i \to j) : (i,j) \in \mathcal{N}, i \neq j, \mathbf{P}_{ij} \leq Y_i^{max}, j \neq source\} \quad (3)$$

The third condition in the right hand side of (3) specifies the set of nodes reachable by a direct transmission from any transmitting node depending on its power constraint. The last condition reflects that no transmitting node needs to reach the source node.

Defining $L_i(t) \triangleq E_i(t)/Y_i$ to be the $lifetime\ of\ node\ i$, the problem of maximizing the TTFF can be written as:

$$maximize \{\min_{i \in \mathcal{N}} L_i(t)\} \quad (4)$$

The objective function in (4) is to be optimized subject to the following constraints:

1) All nodes, other than the source, must be reached, either actually or implicitly[2].
2) The source node must reach at least one other node.
3) The tree must be *connected*; *i.e.*, there must be directed paths from the source to all destination nodes, possibly involving other intermediate nodes.
4) The tree must not have any *cycles*.

The vector $L(t) \triangleq \{L_i(t) : \forall i \in \mathcal{N}\}$ is the $node\ lifetime\ vector$ at time t. Note that the value of the expression within curly braces in (4) is dependent on the time index t and hence, strictly speaking, should be termed $residual-time-to-first-failure$. However, we will refer to it simply as the time-to-first-failure, implicitly recognizing its dependence on the time origin t. Accordingly, henceforth in this paper, we will simply use the notations E_i and L_i instead of $E_i(t)$ and $L_i(t)$.

Assuming that all nodes in the network have omni-directional antennas, a transmission from node i to node j would also be received by all nodes geometrically closer to i than j. Let S be the set of nodes that are geometrically closer to i than j ($\Rightarrow \mathbf{P}_{ij} > \mathbf{P}_{ik} : \forall k \in S$). Nodes that belong in S are said to receive the transmission from i **implicitly** (in the sense that no additional cost is incurred to reach them) and the set of transmissions $\{i \to k : \forall k \in S\}$ are referred to as **implicit transmissions**. The transmission $i \to j$ is referred to as an **actual transmission**.

Let $\{X_{ij} : (i \to j) \in \mathcal{E}\}$ be a set of binary variables such that $X_{ij} = 1$ if the transmission $i \to j$ is used in the optimum tree and 0 otherwise. Following our discussion in the previous paragraph, we can write:

$$Y_i = \max_j \{X_{ij}\mathbf{P}_{ij} : j \neq i\} \quad (5)$$

where $X_{ij} = 1$ if node j is reached from node i (actually or implicitly) and 0 otherwise. Note that equation (5) is a direct consequence of our assumption of omni-directional antennas and implies that the cost of spanning in multiple nodes from node i is simply the cost incurred in reaching the farthest node.

[2]Note that the possibility of reaching a node implicitly is a consequence of the inherently broadcast nature of the wireless network and our assumption of omni-directional antennas.

We now express the objective function in (4) as a minimax optimization problem as shown below:

$$\begin{aligned} & maximize && (\min_i L_i) \\ = & \ maximize && (\min_i E_i/Y_i) \\ = & \ minimize && (\max_i Y_i/E_i) && (6) \\ = & \ minimize && (\max_i [\max_j (\mathbf{P}_{ij}X_{ij})/E_i]) && (7) \\ = & \ minimize && (\max_{i,j} [\mathbf{P}_{ij}X_{ij}/E_i]) && (8) \\ = & \ minimize && \sigma && (9) \end{aligned}$$

where

$$\sigma = \max_{i,j} (\mathbf{P}_{ij}X_{ij}/E_i) = 1/\tau \quad (10)$$

and τ is the TTFF.

We conclude this section with definitions of *critical node* and *critical transmission*. For a given connection tree, T, we define its critical node to be the node whose residual lifetime is equal to the TTFF of the tree. That is:

$$\text{Critical node } = \text{argmin}_i (E_i/Y_i) \quad (11)$$

Note that for any non-transmitting node, $Y_i = 0$, and hence the residual lifetime of that node is ∞.

A transmission $(i \to j)$ is defined to be the critical transmission in a tree if:

$$E_i/\mathbf{P}_{ij} = \text{TTFF } \triangleq \min_i (E_i/Y_i) \quad (12)$$

In the next section, we illustrate with an example the inadequacy of the TTFF criterion when optimized singly. This will motivate the need for a joint objective function involving the sum of transmitter powers. In Section V, we develop a mixed integer linear programming model for the joint optimization problem.

IV. Inadequacy of the TTFF criterion

Consider the 6-node network and the broadcast tree in Figure 1a. Assuming $\alpha = 2$, the power matrix of the network is:

$$\mathbf{P} = \begin{bmatrix} 0 & 14.86 & 9.31 & 6.33 & 7.01 & 1.76 \\ 14.86 & 0 & 23.18 & 4.39 & 4.58 & 6.46 \\ 9.31 & 23.18 & 0 & 7.41 & 24.32 & 11.65 \\ 6.33 & 4.39 & 7.41 & 0 & 7.11 & 2.73 \\ 7.01 & 4.58 & 24.32 & 7.11 & 0 & 2.43 \\ 1.76 & 6.46 & 11.65 & 2.73 & 2.43 & 0 \end{bmatrix} \quad (13)$$

Assume that the residual energy of all nodes is 10. The residual lifetime vector of the nodes for the tree in Figure 1a is: $L_1 = [\infty, 1.55, \infty, 1.35, \infty, 5.69]$. The lifetimes of nodes 1, 3 and 5 are ∞ since they are non-transmitting nodes in the tree. Node 4 is the critical node in the tree and $4 \to 3$ is the critical transmission.

Now consider the broadcast tree in Figure 1b. The residual lifetime vector in this case is: $L_2 =$

$[\infty, 2.28, \infty, 1.35, \infty, \infty]$. The TTFF of this tree is identical to that of Figure 1a. However, note that the lifetime of node 2 is higher (2.28, as compared to 1.55) than its lifetime in Figure 1a. Also, the lifetime of node 6 is now ∞, compared to 5.69 in Figure 1a, since it is a non-transmitting node. Clearly, for the same TTFF, this broadcast tree is better than that shown in Figure 1a.

In general, given two trees T_m and T_n with the same TTFF, T_m is considered better ("leaner") than T_n if:

- there is at least one node in T_m whose residual lifetime is greater in T_m than in T_n, and,
- the residual lifetimes of all other nodes in T_m are at least as high as in T_n.

One way of obtaining a "lean" optimum solution is to consider a joint optimization function of the form:

$$minimize\ \sigma + \beta \left(\sum_{i=1}^{N} Y_i \right) \tag{14}$$

where $\sum_{i=1}^{N} Y_i$ is the sum of transmitter powers, σ is the inverse of the TTFF (10) and β is a suitably chosen non-negative penalty factor. Alternately, we can optimize a convex combination of the two parameters:

$$minimize\ (1-\beta)\sigma + \beta \left(\sum_{i=1}^{N} Y_i \right) \tag{15}$$

It can be easily verified that the tree in Figure 1b is characterized by a smaller total transmitter power, 11.80 units ($P_{24} + P_{43}$), compared to the tree in Figure 1a which uses a total transmitter power of 15.63 units ($P_{26} + P_{61} + P_{43}$).

Note that for $\beta = 0$, an optimal polynomial time algorithm exists, as discussed in [8]. For any $\beta > 0$, however, it is unlikely that any optimal polynomial time algorithm exists, since the problem of minimizing the sum of transmitter powers has been shown to be NP-complete [9].

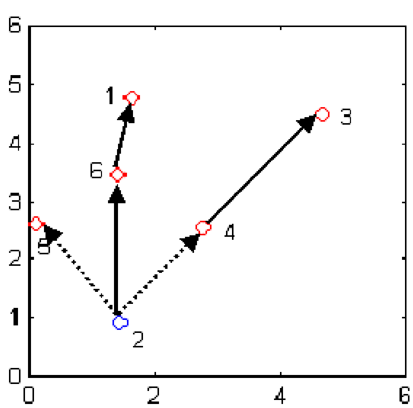

Fig. 1a. The residual energy of each of the nodes is 10. The power matrix of the network is given in (13). TTFF of the broadcast tree $\{2 \to 6, 6 \to 1, 4 \to 3\}$ is 1.35, node 4 being the critical node. The node lifetime vector coresponding to the tree is: $[\infty, 1.55, \infty, 1.35, \infty, 5.69]$. The lifetimes of nodes 1, 3 and 5 are ∞ since they are non-transmitting nodes in the tree. Note that the transmissions $2 \to 4$ and $2 \to 5$ are implicit, since nodes 4 and 5 are nearer to 2 than 6.

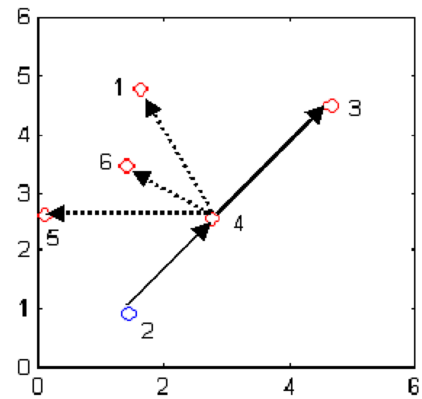

Fig. 1b. An alternate broadcast tree with the same TTFF, 1.35, as in Figure 1a. In this tree, the transmissions $4 \to 1$, $4 \to 5$ and $4 \to 6$ are implicit. The node lifetime vector in this case is: $[\infty, 2.28, \infty, 1.35, \infty, \infty]$. Note that the lifetime of node 2 is higher (2.28, as compared to 1.55) than its lifetime in Figure 1a. Also, the lifetime of node 6 is now ∞, compared to 5.69 in Figure 1a, since it is a non-transmitting node. Clearly, for the same TTFF, this broadcast tree is better than that shown in Figure 1a.

V. MILP Model

We now develop a mixed integer linear programming model of the joint optimization problem involving the TTFF criterion and the sum of the transmitter powers. The objective function can be either of the two forms shown in (14) and (15).

Let $\{F_{ij} : \forall (i \to j) \in \mathcal{E}\}$ be a set of flow variables (F_{ij} represents the flow from node i to node j), with $\mathcal{E}$ defined as in (3). The general multicast problem can be interpreted as a single-origin multiple-destination uncapacitated flow problem, with the source (the *supply node*) having D units of supply and the destination nodes (*demand nodes*) having one unit of demand each. For other nodes, the net in-flow must equal the net out-flow, since they serve only as relay nodes. For example, the broadcast tree in Figure 1b can be represented using the following flow matrix:

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & \mathbf{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

Similarly, if the same tree is used for multicasting to destination nodes 3 and 6, we can write the following flow matrix:

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & 0 & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

At a conceptual level, the flow model can be viewed as a token allocation scheme where the source node generates as many tokens as there are destination nodes and distributes them along the "most efficient" tree such that each destination node gets to keep one token each.

The single-origin multiple-destination flow problem discussed above can be solved using the usual *conservation of flow constraints* as shown below (see for example [10] or [11]):

$$\sum_{j=1}^{N} F_{ij} = D;\ i = source,\ (i \to j) \in \mathcal{E} \tag{18}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 1;\ \forall i \in \mathcal{D}, (i \to j) \in \mathcal{E} \tag{19}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 0;\ \forall i \notin \mathcal{D}, i \neq source, (i \to j) \in \mathcal{E} \tag{20}$$

Having set up the flow equations, we now have to write down constraints linking the flow variables to the power variables, $\{Y_i\}$. We do this in two stages. In the first stage, we couple the flow variables and the indicator variables $\{X_{ij}\}$ and in the next stage, we link the $\{X_{ij}\}$ variables to the power variables. Recall from Section III that $X_{ij} = 1$ if the edge $i \to j$ appears in the optimum solution (either as an actual transmission or as an implicit transmission) and 0 otherwise.

The set of constraints couples the flow variables and the X_{ij} variables:

$$D \cdot X_{ij} - F_{ij} \geq 0;\ \forall (i \to j) \in \mathcal{E} \tag{21}$$

where D is the number of destination nodes. Note that (21) ensures that "$X_{ij} = 1$ if $F_{ij} > 0$". The coefficient of X_{ij} in (21) is due to the fact that the maximum flow out of any node on a single link is equal to the number of destination nodes. Equation (21), however, leaves open the possibility of X_{ij} being equal to 1 for $F_{ij} = 0$. We show later that, for $\beta > 0$ (equations 14 and 15), doing so would unnecessarily increase the cost of the optimum solution and therefore this possibility can be discounted. For the flow matrix in (17), $X_{24} = X_{43} = X_{46} = 1$, the rest of the variables being 0.

Next, we write down constraints linking the X_{ij} variables to the power variables. As discussed in Section III (see eqn. 5), for an omni-directional antenna system, the cost of spanning in multiple nodes from node i is simply the cost incurred in reaching the farthest node. This condition can be expressed as:

$$Y_i - \mathbf{P}_{ij} X_{ij} \geq 0;\ \forall i \in \mathcal{N}, \forall (i \to j) \in \mathcal{E} \tag{22}$$

In order to relate the inverse TTFF parameter, σ, to the power variables, we note that $\sigma = \max_i Y_i/E_i$ (compare equations 6 and 9). As in (22), this condition can be expressed as:

$$\sigma - Y_i/E_i \geq 0;\ \forall i \in \mathcal{N} \tag{23}$$

It is now clear that for $\beta > 0$, if there is no flow out of node i (*i.e.*, $\sum_j F_{ij} = 0$), setting $X_{ij} = 1$ would result in a positive value for Y_i and thereby unnecessarily increase the cost of the optimal solution.

So far, we have implicitly assumed that the residual lifetimes of all transmitting nodes are greater than the multicast duration[3]. In other words, if L is the total number of bits to be transmitted during the session and D is the datarate in bps (assumed uniform throughout the network), we have assumed that:

$$E_i/Y_i \geq L/D \Longleftrightarrow Y_i/E_i \leq D/L \tag{24}$$

Constraints of the form (24) can be explicitly added to the model to ensure that all nodes choose transmitter power levels such that their residual lifetimes are greater than or equal to the multicast session duration (L/D).

The final set of constraints express the integrality of the X_{ij} variables and non-negativity of the F_{ij} and Y_i variables.

$$X_{ij} \geq 0,\ \text{integer};\quad \forall i \in \mathcal{N} \tag{25}$$

$$F_{ij} \geq 0;\quad \forall (i \to j) \in \mathcal{E} \tag{26}$$

$$Y_i \geq 0;\quad \forall i \in \mathcal{N} \tag{27}$$

Figure 2 summarizes the MILP model. We note that the number of integer variables is equal to E while the number of continuous variables is equal to $E + N$. The number of constraints is equal to $2E + 3N$.

VI. Dealing with prioritized nodes

The MILP model we discussed assumes that all nodes enjoy equal priority in the network. We now consider the case where nodes may have unequal priorities, *e.g.*, depending on their location in the grid or on their residual energies. Let w_i be the priority associated with node i, $0 < w_i \leq 1$. The effective lifetime[4] of node i, L_i^{eff}, is now defined as:

$$L_i^{eff} = E_i/w_i Y_i \tag{28}$$

Consequently, we redefine the inverse TTFF parameter as follows (instead of eqn. 10):

$$\sigma = \max_i (w_i Y_i/E_i) = 1/\tau \tag{29}$$

The above equation can be expressed as the following set of linear constraints:

$$\sigma - w_i Y_i/E_i \geq 0;\ \forall i \in \mathcal{N} \tag{30}$$

Solving the optimization problem with (30) instead of (23) yields a node prioritized optimum solution. We illustrate the concept of node weighting with an example.

Consider the 3-node network in Figure 3. Assume $\mathbf{P}_{AB} = 2$, $\mathbf{P}_{BC} = 1.5$, $\mathbf{P}_{AC} = 5$, $E_A = 10$ and $E_B = 5$.

[3] We assume static multicasting; *i.e.*, the same tree is used for the entire multicast duration.

[4] Note that the actual lifetime of node i is still given by E_i/Y_i. The notion of effective lifetime is used only to guide the optimization process to choose routes avoiding the nodes accorded the highest priorities, as illustrated subsequently.

$$minimize\ \sigma + \beta\left(\sum_{i=1}^{N} Y_i\right)$$

or

$$minimize\ (1-\beta)\sigma + \beta\left(\sum_{i=1}^{N} Y_i\right)$$

subject to:

$$\begin{aligned}
\sigma - Y_i/E_i &\geq 0;\ \ \forall i \in \mathcal{N} \\
Y_i/E_i &\leq D/L;\ \ \forall i \in \mathcal{N} \\
Y_i - \mathbf{P}_{ij}X_{ij} &\geq 0;\ \forall i \in \mathcal{N}, \forall(i \to j) \in \mathcal{E} \\
D \cdot X_{ij} - F_{ij} &\geq 0;\ \forall(i \to j) \in \mathcal{E} \\
\sum_{j=1}^{N} F_{ij} &= D;\ i = source,\ (i \to j) \in \mathcal{E} \\
\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} &= 1;\ \forall i \in \mathcal{D}, (i \to j) \in \mathcal{E} \\
\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} &= 0;\ \forall i \notin \mathcal{D}, i \neq source, (i \to j) \in \mathcal{E} \\
X_{ij} &\geq 0,\ \text{integer};\ \ \forall(i \to j) \in \mathcal{E} \\
F_{ij} &\geq 0;\ \ \forall(i \to j) \in \mathcal{E} \\
Y_i &\geq 0;\ \ \forall i \in \mathcal{N}
\end{aligned}$$

Fig. 2. MILP model for the joint inverse TTFF and sum of transmitter powers minimization problem

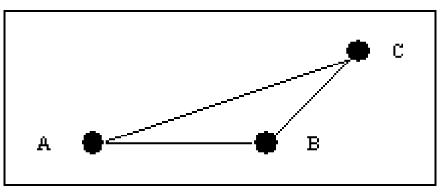

Fig. 3. An example 3-node network.

Let $w_A = w_B = 1$. Under these conditions, the optimal TTFF broadcast tree, considering node A to be the source, is $\{A \to B, B \to C\}$, with a TTFF of 10/3 (node B is the critical node). If, however, $w_A = 0.5$ and $w_B = 1$ (*i.e.*, it is more important to preserve node B than A), it can be easily verified that the optimization process yields the broadcast tree $\{A \to C\}$, with node B reached implicitly. Note that the effective lifetime of node A, as computed by the optimization process, is $E_A/w_A\mathbf{P}_{AC} = 10/(0.5 \times 5) = 4$ but its actual lifetime is $E_A/\mathbf{P}_{AC} = 10/5 = 2$. This example illustrates how node B can be preserved, at the expense of node A, by assigning suitable node weights.

VII. Conclusion

In this paper, we have considered the problem of maximizing the time-to-first-failure in broadcast wireless networks. First, we showed that maximizing the TTFF (or, minimizing the inverse TTFF) criterion, by itself, may not yield the best possible solution. This motivated us to to consider a joint optimization problem involving the sum of transmitter powers. We presented a mixed integer linear programming model for solving the joint optimization problem. We are using the model for benchmarking the performance of fast sub-optimal heuristic algorithms, work on which is currently ongoing. These will be reported in a subsequent paper.

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